CHAPTER VI

CROSS-POWER SPECTRAL ANALYSIS FOR ATMOSPHERIC CROSS-BEAM DATA

by

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VI-1. INTRODUCTION

This is the first attempt to physically interpret cross-beam atmospheric results using spectral analysis. The cross-beam program has used just the time domain for the retrieval of wind velocity information [VI-1, -2]. While the spectral formulation applied to this analysis is not new [VI-3], the method of combining is new.

The concept of a piecewise spectrum is introduced, along with the statistical error of the spectrum. This paper analyzes the above concepts and applies them to actual cross-beam data.

Chapter VII will give the details of how these techniques can be applied to retroactive filtering.

VI-2. NOMENCLATURE

INDEPENDENT VARIABLES

Symbol	Definition
t	total observation time
T	total integration time

Symbol	Definition
ΔT	length of time for each piece -- the whole data record is subdivided into pieces for data processing
i	piece number
m	total number of pieces of data integrated over ($m = T/\Delta T$)
f	frequency
τ	time lag between the two signals
τ_m	maximum time lag

DEPENDENT VARIABLES

$x(t)$	signal time history for the sensor
$y(t-\tau)$	delayed time history for the second sensor
$r_i(\tau)$	covariance of the ith piece of data at time lag τ
$\overline{r_i(\tau)}$	accumulative covariance average over the first ith piece of data
$\overline{\overline{r(\tau)}}$	accumulative covariance average over the entire integration time
$\Delta r(\tau)$	statistical error of the covariance
$C(f)$	co-spectra (real part)
$Q(f)$	quad-spectra (imaginary part)
$G(f)$	gain, $G^2 = C^2 + Q^2$
$\langle G(f) \rangle$	mean gain over m pieces of data
$\overline{\overline{G(f)}}$	accumulative gain over m pieces of data
$\langle \Delta G(f) \rangle$	mean statistical error of the gain
$\overline{\overline{\Delta G(f)}}$	accumulative statistical error of the gain

Symbol	Definition
$r_{G,\phi}$	the Fourier transpose of the spectra to the correlation curve will have the subscripts of those spectra components used
$r_{yx}(\tau) = r_{xy}(-\tau)$	this is the cross-covariance (subscript xy indicates positive time lags and yx indicates negative time lags

OPERATORS

$( )$	statistic
$\overline{( )}_i = \frac{1}{\Delta T} \int_{(i-1)\Delta T}^{\Delta} ( )\, dt$	piecewise mean (note: when average done over real time or time lag, the symbol $< >$ is used
$<( )_k> = \frac{1}{k} \sum_{i=1}^{k} ( )_i$	accumulative mean where k is the kth piece of data (when average over pieces)
$\overline{\overline{( )}} = <\overline{( )}_{k=m}>$	total accumulative mean
$<( )_k>^2 = \frac{1}{k-1} \sum_{i=1} (( )_i - \overline{( )}_k)^2$	statistical error
$<( )> = F\ \{<( )>\}$	the symbols used in the time domain will be transferred into the frequency domain showing the origin of the terms

VI-3. FORMULATIONS

A. Spectra

Two types of spectrum routines are used in this analysis: the accumulative spectra and the mean spectra. The accumulative is obtained from a Fourier transformation of the final accumulative correlation curve,

whereas the mean spectra is obtained by averaging the Fourier transformation of each piece correlation curve. The spectrum obtained by these formulations is not the common method of spectral analysis. Analog and most digital methods of spectral analysis use a fast Fourier transform technique. This means that the spectrum is obtained from the raw data (before correlation of the data). Since we are Fourier-transforming the crosscorrelation curves, we have the spectrum from only the common modulations of the two signals. Therefore, for large noise, the advantage is that most of the noise is cancelled out before the spectrum is taken.

The formulation used to obtain the accumulative cospectra $\overline{\overline{C(f)}}$ is

$$\overline{\overline{C(f)}} = \int_0^{\tau_m} [\overline{\overline{r_{xy}(\tau)}} + \overline{\overline{r_{yx}(\tau)}}] \cos 2\pi f\tau \, d\tau, \tag{1}$$

where τ is the delay, $\overline{\overline{r(\tau)}}$ is the accumulative covariance, and f is the frequency. The accumulative quad-spectrum $\overline{\overline{Q(f)}}$ is

$$\overline{\overline{Q(f)}} = \int_0^{\tau_m} [\overline{\overline{r_{xy}(\tau)}} - \overline{\overline{r_{yx}(\tau)}}] \sin 2\pi f\tau \, d\tau. \tag{2}$$

It then follows that the modulus of the accumulative gain $\overline{\overline{G(f)}}$ is

$$\overline{\overline{G(f)}} = \sqrt{\overline{\overline{C(f)}}^2 + \overline{\overline{Q(f)}}^2} \tag{3}$$

and the accumulative phase $\overline{\overline{\phi(f)}}$ is

$$\overline{\overline{\phi(f)}} = \arctan \frac{\overline{\overline{Q(f)}}}{\overline{\overline{C(f)}}} . \tag{4}$$

These values given by the last four equations are calculated in the computer program given in the appendix.

The values for the piecewise gain $\bar{G}_i$, co-spectra $\bar{C}_i$, and quad-spectra $\bar{Q}_i$ are calculated in exactly the same manner as their accumulative counterparts except that the covariance curve used is now the piecewise covariance curve. With these terms, the mean gain can be defined as

$$\overline{\overline{\langle G_i(f) \rangle}} = \frac{1}{m} \sum \overline{\overline{G_i(f)}}. \tag{5}$$

The mean gain is a unique term not normally found in the literature. There are reasons to believe that the mean gain is free from small oscillations in time delay.

After the error criteria are considered, these two gains will be tested on actual data.

B. Statistical Spectra Error

One of the key problems in spectral analysis is the lack of a good error routine. If one deals with either the co-spectra or the quad-spectra, then the standard error routines give unique results. However, this is not true for either the gain or the phase, since these errors depend on (a) which average (accumulative or mean) is used, and (b) the differences in error propagation with changes in integration time and summation.

The statistical error of the accumulative gain $\overline{\overline{\Delta G(f)}}$ is defined as

$$\overline{\overline{\Delta G(f)}} = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} [\overline{G_i(f)} - \overline{\overline{G(f)}}]^2}, \tag{6}$$

and it follows then that the statistical error of the mean gain $\langle \overline{\Delta G(f)} \rangle$ is defined as

$$\langle \overline{\Delta G(f)} \rangle = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} [\overline{G_i(f)} - \langle \overline{G(f)} \rangle]^2}. \tag{7}$$

These error calculations were performed using the computer program given in the appendix.

C. Inverse Transform of Spectra

The next question is, what happens when the spectrum is transposed back into the time domain? This, in effect, does provide a check on the work done in the frequency domain.

Two types of inverse transforms exist: a transpose of the co-spectra and quad-spectra, and a transpose of the gain and the phase. The component transpose is given by

$$r_{\underset{CQ}{==}}(\tau) = \int_{f_{min}}^{f_{max}} [\overline{\overline{C(f)}} \cos (2\pi f\tau) - \overline{\overline{Q(f)}} \sin (2\pi f\tau)] \, dt, \qquad (8)$$

and the gain transpose is given by

$$r_{\underset{<\bar{G}>\emptyset}{\bar{=}}}(\tau) = \int_{f_{min}}^{f_{max}} \overline{<G(f)>} \cos [2\pi f\tau + \overline{\overline{\emptyset(f)}}] \, df. \qquad (9)$$

The subscripts indicate the source of the terms used in the transpose. The computer program for the above calculations is given in the appendix.

VI-4. RESULTS

The formulations discussed in the preceding section will be tested on actual atmospheric data from a separated cross-beam run with high statistical accuracy to perhaps gain a reasonable empirical understanding of the spectra.

The first formulation calculated was the accumulative gain (equation (3)) and its associated error (equation (6)). The results are shown in the top graph of Figure VI.1. These results show that the accumulative gain is not statistically significant over the range of frequencies from 0.01 cps to 1.0 cps. We have required only about a 65 percent level of probability in the test of significance. Since this run contains a peak in the correlation curve with a statistical probability of better than 99 percent, the spectra from other curves are going to be even less significant. To check for errors, the spectrum was transposed back to the time domain by equation (8). These results are shown in Figure VI.2. One observes that the transpose of this spectrum gives the initial correlation curve; therefore, the accumulative spectrum must have the same information in it that was contained in the original curve.

Before drawing any conclusions we tried another spectral routine. Rather than using the accumulative spectrum, we used the mean spectra (equation (5)) with its associated error (equation (7)). The results are shown in the lower portion of Figure VI.1. That the mean gain is statistically significant suggests that perhaps we are on the right track. To verify these results, the mean gain and the mean phase were transposed back to the time domain by equation (9). These results, shown in Figure VI.3, appear to be bad, but the mean gain is approximately the same as the accumulative gain, according to Figure VI.1. Thus, this result seemed to be only an effect of the variations in phase. This theory was tested by transposing both the mean gain and accumulative phase, and the accumulative gain and mean phase (see Figures VI.4 and VI.5). The correlation curve obtained from the mean gain and the accumulative phase (Figure VI.4) gives the best correlation curve thus far obtained, and the accumulative gain and the mean phase (Figure VI.5) give about the same results as were obtained for the mean gain and the mean phase (Figure VI.3).

VI-5. CONCLUSIONS

Thus far, the spectral results obtained have been analyzed only for the most statistically significant atmospheric data we have. Based on the empirical results just given, the following tentative conclusions are reached.

The computation of covariance functions from an inverse Fourier transform of piecewise-estimated phase and gain spectra may deviate from the direct product integration, if the phase spectrum is taken from an accumulation of piecewise averages. A reasonable explanation for this observation is that the mean gain cancels the effects of nonstationarities in transit times that are normally reflected in changes in the phase. Thus, evaluating the piecewise correlation curve in the frequency domain, where shifts in the peak can be neglected, and then transposing the final mean result back into time domain may prove to be a better technique for dealing with nonstationarities than just normalizing each piecewise correlation curve.

Another tentative conclusion is that error criteria can be established only after plotting the error curves for both accumulative and mean gain and phase, and comparing the absolute error with the relative error. The present error curves should allow the separation of environmental variations of gain and phase.

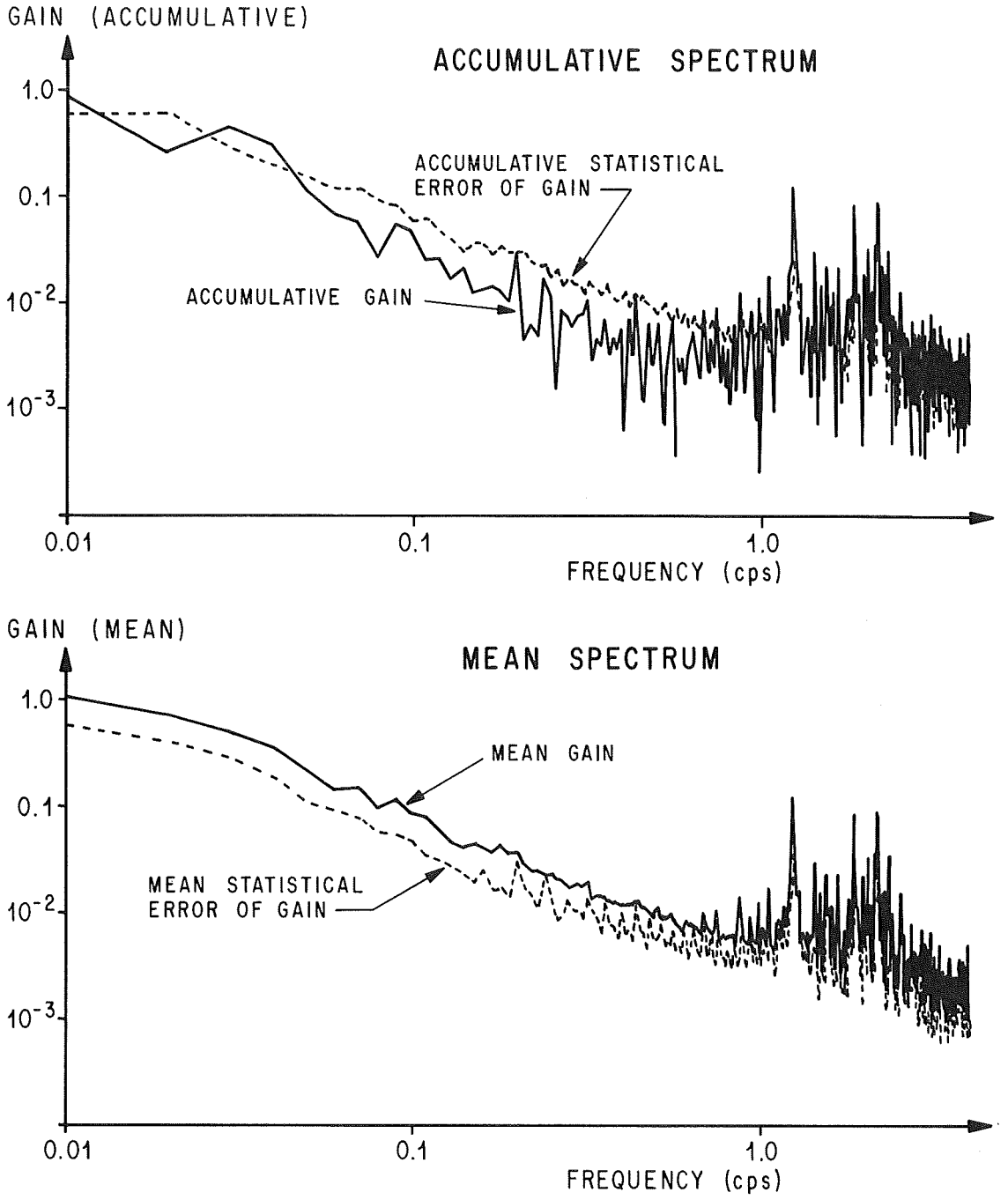

FIGURE VI.1. EXCHANGED ORDER OF ACCUMULATION AND SPECTRUM CALCULATIONS

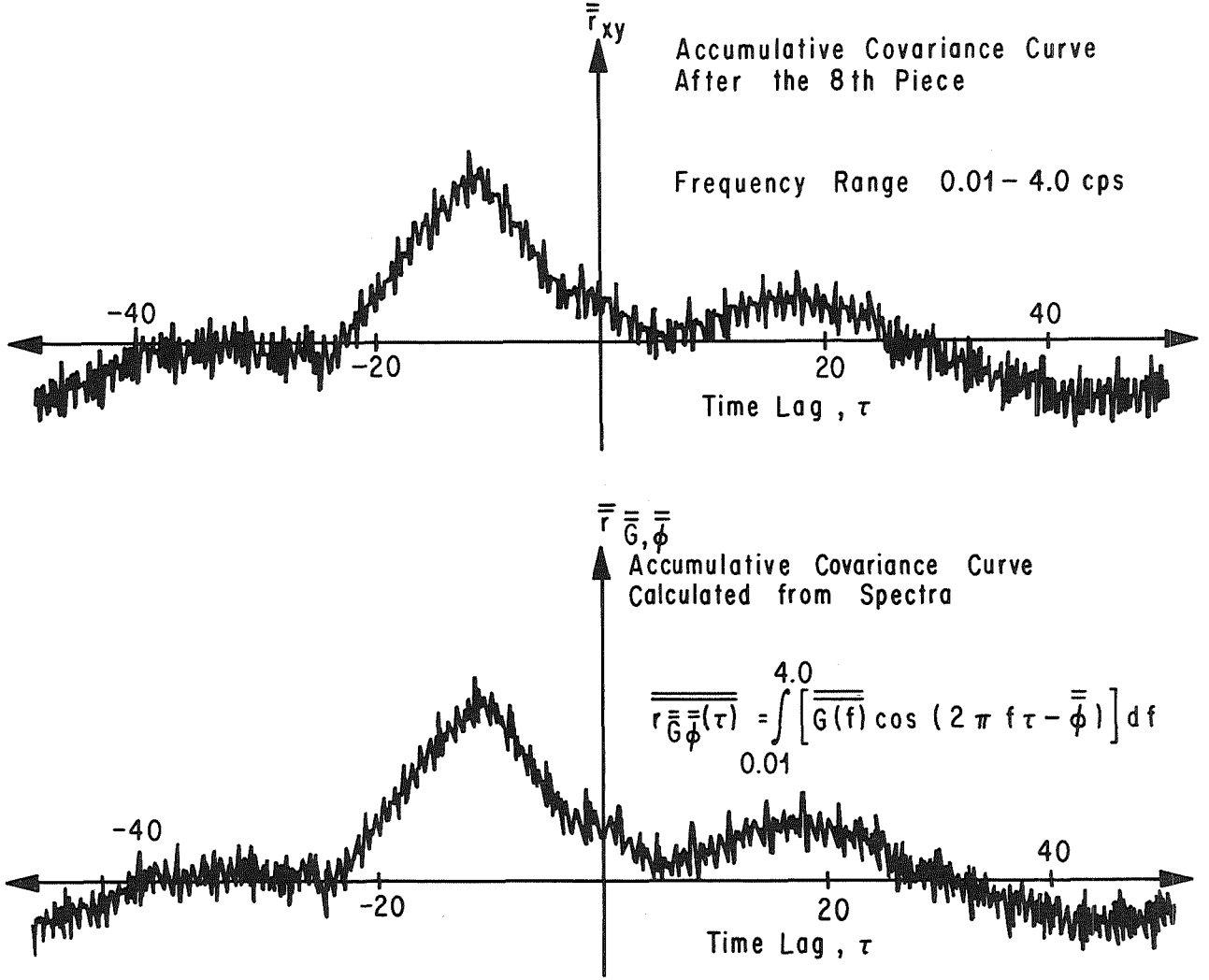

FIGURE VI.2. DIRECT AND DOUBLE-TRANSFORMED ACCUMULATIVE CORRELATION CURVES

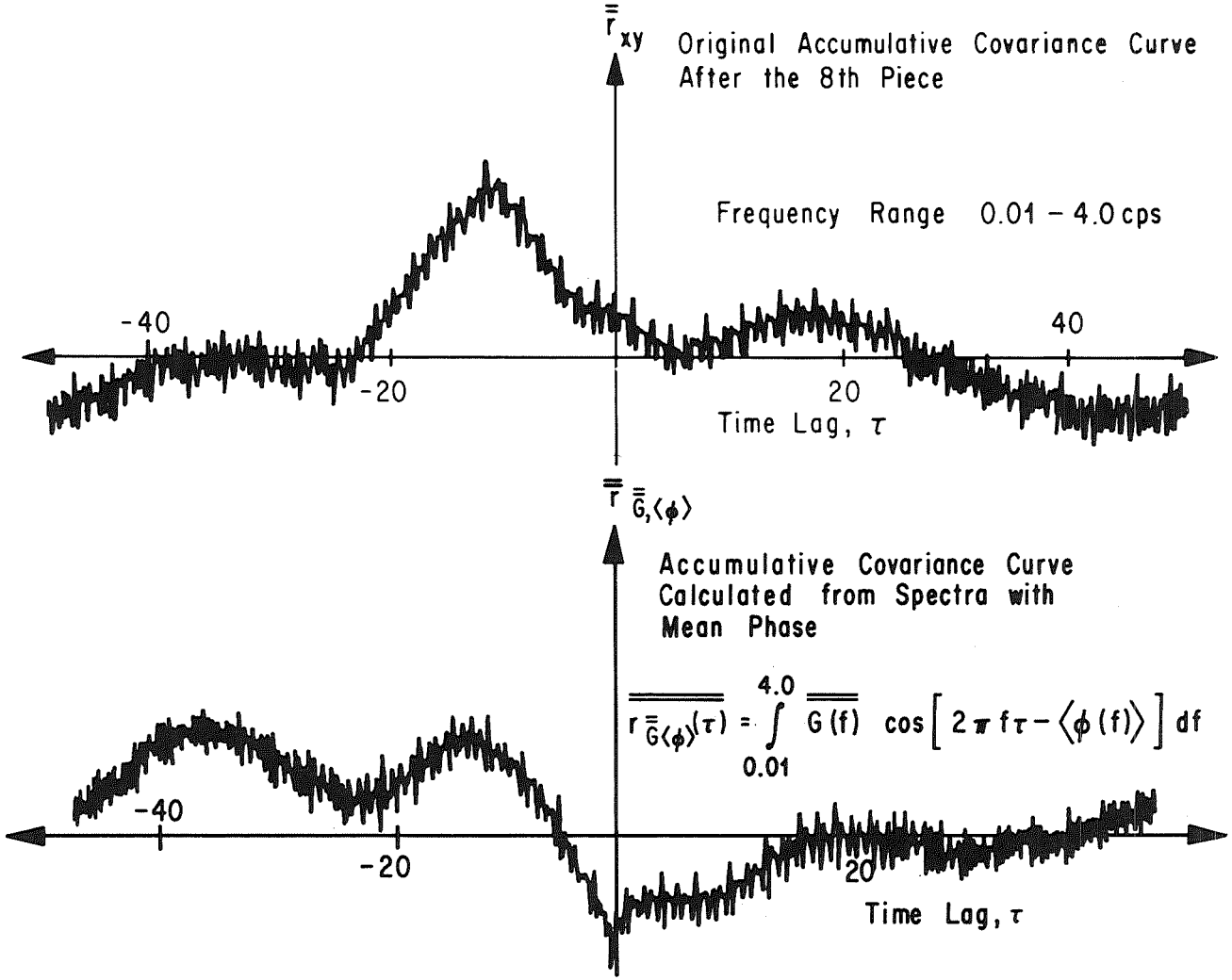

FIGURE VI.3. DIRECT AND DOUBLE-TRANSFORMED ACCUMULATIVE CORRELATION CURVES

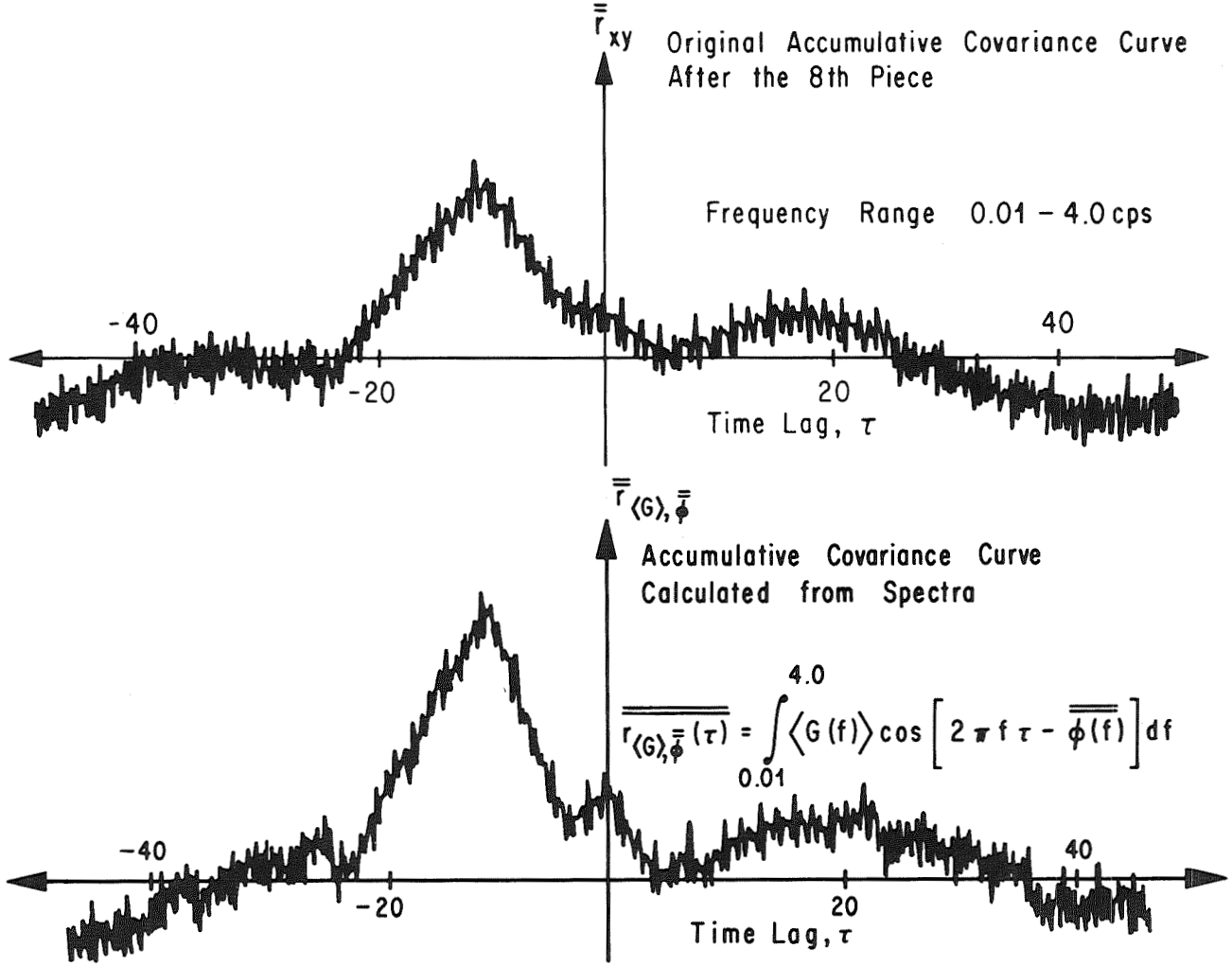

FIGURE VI.4. DIRECT AND DOUBLE-TRANSFORMED ACCUMULATIVE CORRELATION CURVES

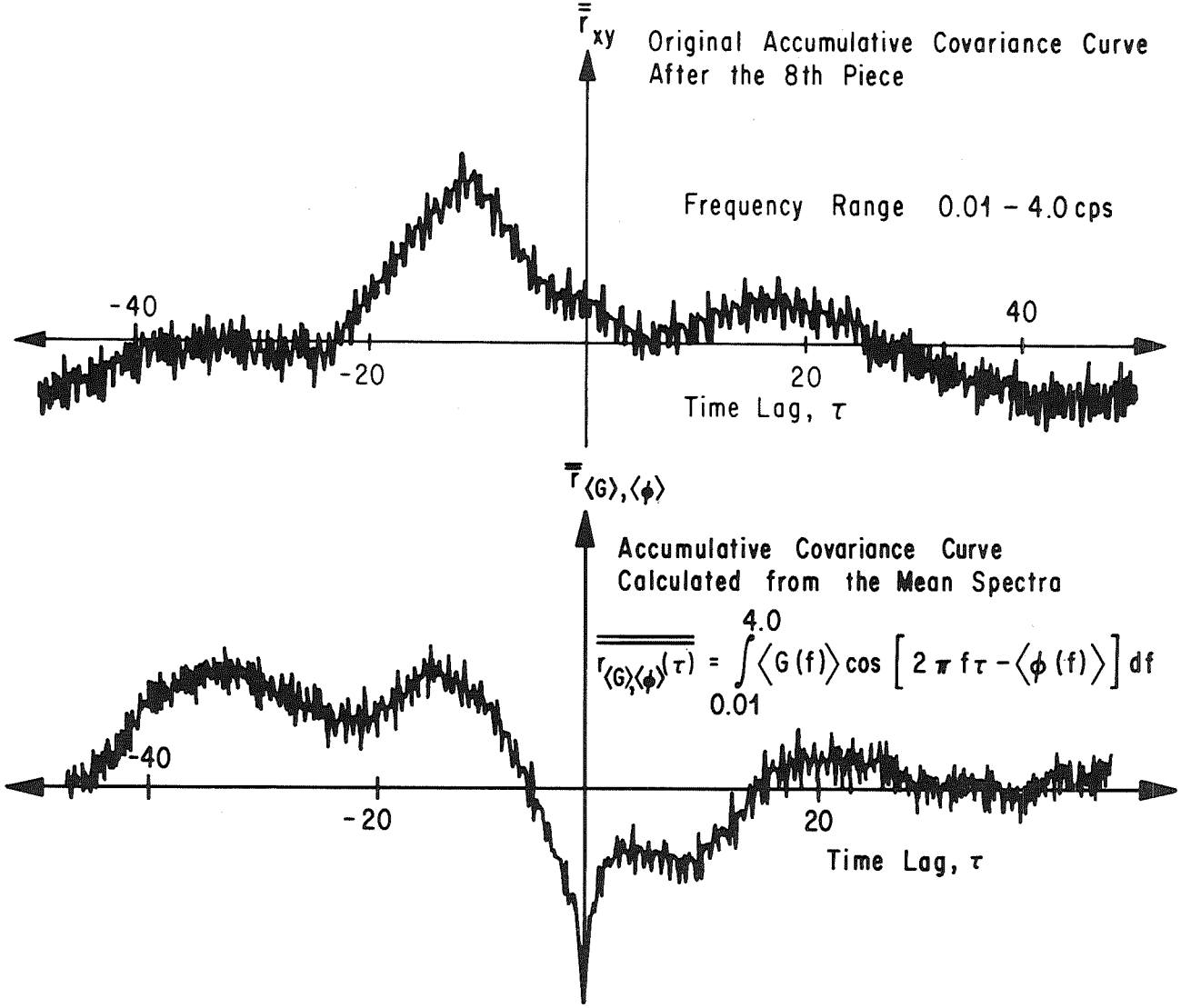

FIGURE VI.5. DIRECT AND DOUBLE-TRANSFORMED ACCUMULATIVE CORRELATION CURVES

REFERENCES

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